CONFIDENTIAL

UNIVERSITY COLLEGE TATI

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET	
COURSE CODE	: DGE 2123
COURSE	: MATHEMATICS II
SEMESTER/SESSION	: 1-2021/2022 & 1-2022/2023
DURATION	: 3 HOURS

Instructions:

1. This booklet contains **5** questions in SECTION A, **3** questions in SECTION B and **2** questions in SECTION C. Answer **ALL** questions.
2. All answers should be written in answer booklet.
3. Write legibly and draw sketches wherever required.
4. If in doubt, raise your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 6 PRINTED PAGES INCLUDING COVER PAGE

SECTION A (50 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

Differentiate each of the following function with respect to x .

a) $y = 4x^3 - 5x^2 + 6x + 5$ (2 marks)

b) $y = 8e^{5x}$ (2 marks)

c) $y = -5\cos 3x$ (2 marks)

d) $y = (7x + 12)^3$ (2 marks)

e) $y = \ln(2x + 1)$ (2 marks)

QUESTION 2

Integrate the following indefinite integrals.

a) $\int (5x^4 + 3x)\,dx$ (2 marks)

b) $\int \frac{2}{x-5}\,dx$ (2 marks)

c) $\int (6\sin x + 2)\,dx$ (2 marks)

d) $\int (e^{3x} - 3x^2)\,dx$ (2 marks)

e) $\int \frac{1}{5x+2}\,dx$ (2 marks)

QUESTION 3

Find the derivatives for the following function using the given techniques.

a) $f(x)=(3x^2-4x)e^{-3x}$ (Use product rule) (3 marks)

b) $f(x)=\dfrac{x^3-2}{\ln x}$ (Use quotient rule) (3 marks)

QUESTION 4

Integrate the following functions.

a) $\int 3(x^2-1)e^{x^3-3x}\,dx$ (Substitution method) (4 marks)

b) $\int 2x\ln x\,dx$ (By Parts method) (4 marks)

c) $\int \dfrac{3x+5}{(2x+1)(x-3)}dx$ (Partial Fraction method) (7 marks)

QUESTION 5

Find the general solution for the following differential equations.

a) $y''-6y'+8y=0$ (3 marks)

b) $y''+2y'+y=0$ (3 marks)

c) $y''+3y'+7y=0$ (3 marks)

SECTION B (30 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

a) Find $\frac{dy}{dx}$ for $3x^2 + 4y^2 = 15xy - 6$ using implicit differentiation. (6 marks)

b) Find $\frac{dy}{dx}$ given that $x = t^3 + 3t^2$ and $y = t^4 - 8t^2$. (4 marks)

QUESTION 2

a) Solve the following differential equation by separation of variables method.

$$x\frac{dy}{dx} = y + 2x^2 y$$

(4 marks)

b) Solve the following differential equation using an integrating factor method.

$$x^2\frac{dy}{dx} + xy = 1$$

(6 marks)

QUESTION 3

Solve the given differential equation by using method of undetermined coefficient.

$$y'' - 6y' + 5y = -9e^{2x}$$

(10 marks)

SECTION C (20 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

Given $f(x) = x^3 - 3x^2 - 9x + 6$,

a) Find the stationary points on the curve. (4 marks)

b) Determine their nature. (4 marks)

c) Sketch the curve. (2 marks)

QUESTION 2

Find the area enclosed by the curve $y = x^2 - 30$ and straight line $y = 10 - 3x$. Hence, sketch the curve and label the shaded the area.

(10 marks)

……………………………END OF QUESTIONS……………………………

FORMULA

$\frac{d}{dx}(k)=0$	$\int(ax+b)^n dx=\frac{(ax+b)^{n+1}}{(n+1)(a)}+C,\ \text{for } n\neq -1$
$\frac{d}{dx}(x^n)=nx^{n-1}$	$\int\cos(ax+b)\,dx=\frac{1}{a}\sin(ax+b)+C$
$\frac{d}{dx}(kx^n)=knx^{n-1}$	$\int\sin(ax+b)\,dx=\frac{-1}{a}\cos(ax+b)+C$
$\frac{d}{dx}[f(x)]^n=n[f(x)]^{n-1}f'(x)$	$\int\frac{1}{ax+b}dx=\frac{1}{a}\ln\|ax+b\|+C$
$\frac{d}{dx}(\sin u)=u'\cos u$	$\int e^{ax+b}dx=\frac{1}{a}e^{ax+b}+C$
$\frac{d}{dx}(\cos u)=-u'\sin u$	$\int udv=uv-\int vdu$
$\frac{d}{dx}(\sec u)=u'\sec u\tan u$	$A=\int_a^b[f(x)-g(x)]dx$
$\frac{d}{dx}[e^u]=u'e^u$	$\frac{dy}{dx}=m(x)\,n(y)$
$\frac{d}{dx}[\ln u]=\frac{1}{u}\cdot u'$	$\frac{dy}{dx}+P(x)y=Q(x)$
$\frac{d}{dx}(uv)=uv'+vu'$	$V(x)=e^{\int P(x)dx}$
$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{vu'-uv'}{v^2}$	$V(x)y=\int Q(x)V(x)\,dx$
$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$	$y_c=k_1e^{mx}+k_2xe^{mx}$
$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$	$y_c=k_1e^{m_1x}+k_2e^{m_2x}$
$\int kdx=kx+C$	$y_c=e^{ax}(k_1\cos bx+k_2\sin bx)$
$\int x^n dx=\frac{x^{n+1}}{n+1}+C,\ \text{for } n\neq -1$	$y=y_c+y_p$

$g(x)$	y_p
$ax^r++bx+c$	$Ax^r+........+Bx+C$
ae^{cx}	Ae^{cx}